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SUBCANONICAL GRADED RINGS WHICH ARE NOT COHEN-MACAULAY

FABRIZIO CATANESE

This article is dedicated to Rob Lazarsfeld on the occasion of his 60*-th birthday.*

Abstract. We answer a question by Jonathan Wahl, giving examples of regular surfaces (so that the canonical ring is Gorenstein) with the following properties:

1) the canonical divisor $K_S \equiv rL$ is a positive multiple of an ample divisor L

2) the graded ring $\mathcal{R} := \mathcal{R}(X, L)$ associated to L is not Cohen-Macaulay.

In the appendix Wahl shows how these examples lead to the existence of Cohen-Macaulay singularities with K_X $\mathbb{Q}$-Cartier which are not $\mathbb{Q}$-Gorenstein, since their index one cover is not Cohen-Macaulay.

Contents

1. Introduction

The situation that we shall consider in this paper is the following: L is an ample divisor on a complex projective manifold X of complex dimension n, and we assume that L is subcanonical, i.e., there exists an integer h such that we have the linear equivalence $K_X \equiv hL$, where $h \neq 0$.

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There are then two cases: $h < 0$, and X is a Fano manifold, or $h > 0$ and X is a manifold with ample canonical divisor (in particular X is of general type).

Assume that X is a Fano manifold, and that $-K_X = rL$, with $r > 0$: then, by Kodaira vanishing

$$H^j(mL) := H^j(\mathcal{O}_X(mL)) = 0 \; \forall m \in \mathbb{Z}, \forall 1 \leq j \leq n-1.$$

For $m < 0$ this follows from Kodaira vanishing (and holds for $j \geq 1$), while for $m \geq 0$ Serre duality gives $h^j(mL) = h^{n-j}(K - mL) = h^{n-j}((-r-m)L) = 0$.

At the other extreme, if K_X is ample, and $K_X \equiv rL$, (thus $r > 0$) by the same argument we get vanishing outside of the interval

$$0 \leq m \leq r.$$

To L we associate as usual the finitely generated graded $\mathbb{C}$-algebra

$$\mathcal{R}(X, L) := \oplus_{m \geq 0} H^0(X, \mathcal{O}_X(mL))$$

Therefore in the Fano case, the divisor L is arithmetically Cohen-Macaulay (see [Hart77]) and the above graded ring is a Gorenstein ring.

The question is whether also in the case where K_X is ample one may hope for such a good property.

The above graded ring is integral over the canonical ring $\mathcal{A} := \mathcal{R}(X, K_X)$, which is a Gorenstein ring if and only if we have *pluri-regularity*, i.e., vanishing

$$H^j(\mathcal{O}_X) = 0 \; \forall \; 1 \leq j \leq n-1.$$

Jonathan Wahl asked the following question (which makes only sense for $n \geq 2$):

Question 1. (J. Wahl) *Are there examples of subcanonical pluriregular varieties X such that the graded ring $\mathcal{R}(X, L)$ is not Cohen-Macaulay ?*

We shall show that the answer is positive, also in the case of regular subcanonical surfaces with K_X ample, where by the assumption we have the vanishing

$$H^1(mL) = 0 \; \forall m \leq 0, \text{ or } r \leq m$$

and the question boils down to requiring the vanishing also for $1 \leq m \leq r-1$.

The following theorem answers the question by J. Wahl:

Theorem 2. *For each $r = n-3$, where $n \geq 7$ is relatively prime to 30, and for each m, $1 \leq m \leq r-1$ there are Beauville type surfaces S with $q(S) = 0$ ($q(S) := \dim H^1(S, \mathcal{O}_S)$) s.t. $K_S = rL$, and $H^1(mL) \neq 0$.*

We get therefore examples of the following situation: $\mathcal{A} := \mathcal{R}(S, K_S)$ is a Gorenstein graded ring, and a subring of the ring $\mathcal{R} := \mathcal{R}(S, L)$, which is not arithmetically Cohen-Maculay; hence we have constructed examples of non Cohen-Macaulay singularities (Spec ($\mathcal{R}$)) with K_Y Cartier which are cyclic quasi-étale covers of a Gorenstein singularity (Spec ($\mathcal{A}$)).

In the Appendix, J. Wahl uses these to construct Cohen-Macaulay singularities with K_X $\mathbb{Q}$-Cartier whose index one cover is not Cohen-Macaulay.

In fact, we can consider three graded rings, two of which are subrings of the third, and which are cones associated to line bundles on the surface S:

- $Y :=$ Spec ($\mathcal{R}$), the cone associated to L, which is not Cohen-Macaulay, while K_Y is Cartier;
- $Z :=$ Spec ($\mathcal{A}$), the cone associated to K_S, which is Gorenstein;
- $X :=$ Spec ($\mathcal{B}$), the cone associated to $K_S + L$ (for instance), which is Cohen-Macaulay with K_X $\mathbb{Q}$–Cartier, but whose index 1 (or canonical) cover $Y =$ Spec ($\mathcal{R}$) is not Cohen-Macaulay.

2. The special case of even surfaces

Recall: a smooth projective surface S is said to be **even** if there is a divisor L such that $K_S \equiv 2L$.

This is a topological condition, it means that the second Stiefel Whitney class $w_2(S) = 0$, or, equivalently, the intersection form

$$H^2(S, \mathbb{Z}) \to \mathbb{Z}$$

is even (takes only even values).

In particular, an even surface is a minimal surface.

In particular, if S is of general type and even, the self intersection

$$K_S^2 = 4L^2 = 8k$$

for some integer $k \geq 1$.

The first numerical case is therefore the case $K_S^2 = 8$.

Proposition 3. *Assume that S is an even surface of general type with $K_S^2 = 8$ and $p_g(S) = h^0(K_S) = 0$. Then, if $K_S \equiv 2L$, then $H^1(L) = 0$.*

Proof. We assume that S is even, $K \equiv 2L$, and $p_g = 0$.

Since the intersection form is even, and $K^2 \leq 9$ by the Bogomolov-Miyaoka - Yau inequality, we obtain that $L^2 = 2$.

The Riemann Roch theorem tells us: $\chi(L) = 1 + \frac{1}{2}L(L-K) = 1 + \frac{1}{2}L(-L) = 0$.

On the other hand, by Serre duality $\chi(L) = 2h^0(L) - h^1(L)$, so if $H^1(L)$ is different from zero, then $H^0(L) \neq 0$, contradicting $p_g = 0$. □

Our construction for $n = 5$ shall show in particular that the 'Beauville surface', constructed by Beauville in [Bea78] is an even surface with $K_S^2 = 8, q(S) = p_g(S) = 0$, but with $H^1(L) = 0$.

3. Canonical linearization on Fermat curves

Fix a positive integer $n \geq 5$, and let C be the degree n Fermat curve

$$C := \{(x, y, z) \in \mathbb{P}^2 | f(x, y, z) := x^n + y^n + z^n = 0\}.$$

Let as usual μ_n be the group of n-roots of unity.

The the group

$$G := \mu_n^2 = \mu_n^3 / \mu_n$$

acts on C, and we obtain a natural linearization of $\mathcal{O}_C(1)$ by letting $(\zeta, \eta) \in \mu_n^2$ act as follows:

$$z \mapsto z, x \mapsto \zeta x, y \mapsto \eta y.$$

In other words, $H^0(\mathcal{O}_C(1))$ splits as a direct sum of one dimensional eigenspaces (respectively generated by x, y, z) corresponding to the characters $(1, 0), (0, 1), (0, 0) \in (\mathbb{Z}/n)^2 \cong \mathrm{Hom}(G, \mathbb{C}^*)$.

Similarly, for $m \leq n - 1$, the monomial $x^a y^b z^{m-a-b} \in H^0(\mathcal{O}_C(m))$ generates the unique eigenspace for the character (a, b) (we identify here $\mathbb{Z}/n \cong \{0, 1, \dots, n-1\}$ and we obviously require $a + b \leq m$).

However, any two linearizations differ (see [Mum70]) by a character of the group.

Definition 4. *Assume that n is not divisible by 3.*

We call the **canonical** *linearization on $H^0(\mathcal{O}_C(1))$ the one obtained from the natural one by twisting with the character $(n-3)^{-1}(1, 1)$. Thus x corresponds to the character $v_1 := (1, 0) + (n-3)^{-1}(1, 1) = (-3)^{-1}(-2, 1)$, y corresponds to the character $v_2 := (0, 1) + (n-3)^{-1}(1, 1) = (-3)^{-1}(1, -2)$, z corresponds to the character $v_3 := (-3)^{-1}(1, 1)$.*

Remark 5. (I) Observe that v_1, v_2 are a basis of $(\mathbb{Z}/n)^2$ as soon as n is not divisible by 3.

Indeed, $v_1 + v_2 = \frac{1}{3}(1, 1) = 3^{-1}(1, 1)$, hence

$$(1, 0) = v_1 + 3^{-1}(1, 1) = 2v_1 + v_2, \quad (0, 1) = 2v_2 + v_1.$$

(II) Observe that the above linearization induces a linearization on all multiples of L, and, in the case where $m = (n - 3)$, we obtain the natural linearization on the canonical divisor of C, $\mathcal{O}_C(n-3) \cong \Omega_C^1$.

Since, if we take affine coordinates where $z = 1$, and we let f the equation of C, we have

$$H^0(\Omega_C^1) = \{P(x, y) \frac{dx}{f_y} = -P(x, y) \frac{dy}{f_x}\}$$

and the monomial $P = x^a y^b$ corresponds under this isomorphism to the character $(a + 1, b + 1)$.

(III) In particular, Serre duality

$$H^0(\mathcal{O}_C(m)) \times H^1(\Omega^1_C(-m)) \to H^1(\Omega^1_C) \cong \mathbb{C},$$

where $\mathbb{C}$ is the trivial G-representation, is G-invariant.

From the previous discussion follows also

Lemma 6. *The monomial $x^a y^b z^c \in H^0(\mathcal{O}_C(m))$ (here $a, b, c \geq 0$, $a + b + c = m$) corresponds to the character χ equal to:*

$$(a, b) + (-3)^{-1}(m, m) = (a - c)v_1 + (b - c)v_2.$$

Proof. $v_1 + v_2 = \frac{1}{3}(1, 1)$, hence $(a, b) + (-3)^{-1}(m, m) = av_1 + bv_2 + (-m + a + b)(3)^{-1}(1, 1) = (a - c)v_1 + (b - c)v_2$.

□

4. Abelian Beauville Surfaces and their subcanonical divisors

We recall now the construction (see also [Cat00], or [BCG05]) of a Beauville surface with Abelian group $G \cong (\mathbb{Z}/n)^2$, where n is not divisible by 2 and by 3.

Definition 7. *(1) Let $\Sigma \subset G$ be the union of the three respective subgroups generated by* $(1, 0), (0, 1), (1, 1)$.

(2) Let $\psi : G \to G$ a homomorphism such that, setting $\Sigma^* := \Sigma \setminus \{(0, 0)\}$, $\psi(\Sigma^*) \cap \Sigma^* = \emptyset$ (equivalently, $\psi(\Sigma) \cap \Sigma = \{(0, 0)\}$).

(3) Let C be the degree n Fermat curve and let

$$S = (C \times C)/(\mathrm{Id} \times \psi)(G),$$

i.e., the quotient of $C \times C$ by the action of G such that $g(P_1, P_2) = (g(P_1), \psi(g)(P_2))$.

Remark 8. (i) By property (2) G acts freely and S is a projective smooth surface with ample canonical divisor.

(ii) The line bundle $\mathcal{O}_{C\times C}(1, 1)$ is $G \times G$ linearized, in particular it is $G \cong (\mathrm{Id} \times \psi)(G)$-linearized, therefore it descends to S, and we get a divisor L on S such that the pull back of $\mathcal{O}_S(L)$ is the above G-linearized bundle.

(iii) By the previous remarks, we have a linear equivalence

$$K_S \equiv (n - 3)L.$$

5. Cohomology of multiples of the subcanonical divisor L

We consider now an integer m with

$$1 \leq m \leq n - 4,$$

and we shall determine the space $H^1(\mathcal{O}_S(mL))$.
Observe first of all that $H^1(\mathcal{O}_S(mL)) \cong H^1(\mathcal{O}_{C\times C}(m, m))^G$.

By the Künneth formula

$$H^1(\mathcal{O}_{C\times C}(m,m)) \cong$$

$$\cong [H^0(\mathcal{O}_C(m)) \otimes H^1(\mathcal{O}_C(m))] \bigoplus [H^1(\mathcal{O}_C(m)) \otimes H^0(\mathcal{O}_C(m))].$$

We want to decompose the right hand side as a representation of $G \cong (\mathrm{Id} \times \psi)(G)$.

Explicitly, $H^0(\mathcal{O}_C(m)) = \oplus_\chi V_\chi$, where if we write the character $\chi = (a,b) + (-3)^{-1}(m,m)$ ($\chi = (a-(m-a-b))v_1 + (b-(m-a-b))v_2$ as we saw) then V_χ has dimension equal to one and corresponds to the monomial $x^a y^b z^{m-a-b}$, where $a,b \geq 0,\ a+b \leq m$.

By Serre duality, $H^1(\mathcal{O}_C(m)) = \oplus_{\chi'} V_{-\chi'}$, where if we write as above $\chi' = (a',b') + (-3)^{-1}(m',m')$, then $V_{-\chi'}$ is the dual of $V_{\chi'}$, corresponding to the monomial $x^{a'} y^{b'} z^{m'-a'-b'}$, where $m' = n-3-m$, so $1 \leq m' \leq n-4$ also, and where $a',b' \geq 0,\ a'+b' \leq m'$.

Now, the homomorphism $\psi : G \to G$ induces a dual homomorphism $\phi := \psi^\vee : G^\vee \to G^\vee$, therefore we can finally write $H^1(\mathcal{O}_{C\times C}(m,m))$ as a representation of $G \cong (\mathrm{Id} \times \psi)(G)$:

$$H^1(\mathcal{O}_{C\times C}(m,m)) = \bigoplus_{\chi,\chi'} [(V_\chi \otimes V_{-\phi(\chi')}) \oplus (V_{-\chi'} \otimes V_{\phi(\chi)})].$$

We have proven therefore the

Lemma 9. *$H^1(\mathcal{O}_S(mL)) \neq 0$ if and only if there are characters $\chi = (a-c)v_1 + (b-c)v_2$ and $\chi' = (a'-c')v_1 + (b'-c')v_2$ with $a,b \geq 0,\ a+b \leq m,\ a',b' \geq 0,\ a'+b' \leq m' = n-3-m$ such that*

$$\chi = \phi(\chi') \text{ or } \chi' = \phi(\chi).$$

Proof of theorem 2.

We take now ϕ to be given by a diagonal matrix in the basis v_1, v_2, i.e., such that

$$\phi(v_j) = \lambda_j v_j,\ j = 1,2,\ \lambda_j \in (\mathbb{Z}/n)^*.$$

For further use we also set $\lambda := \lambda_1, \mu := \lambda_2$.

Given n relatively prime to 30 and $1 \leq m \leq n-4$, we want to find λ_1 and μ such that the equations

$$(a-c) = \lambda(a'-c')$$
$$(b-c) = \mu(b'-c')$$

have solutions with $a,b,c \geq 0,\ a+b+c = m$, and $a',b',c' \geq 0$, $a'+b'+c' = m'$.

The first idea is simply to take $b = c$ and $b' = c'$, so that μ can be taken arbitrarily.

For the first equation some care is needed, since we want that λ be a unit: for this it suffices that $(a-c), (a'-c')$ are both units, for instance they could be chosen to be equal to one of the three numbers $1, 2, 3$, according to the congruence class of m, respectively m', modulo 3.

With this proviso we have to verify that we have a free action on the product.

Lemma 10. *If $n \geq 7$, given λ a unit, there exists a unit μ such that $\psi = \phi^\vee$ satisfies the condition $\psi(\Sigma) \cap \Sigma = \{(0,0)\}$.*

Proof. Since $(1,0) = 2v_1 + v_2$ and $(0,1) = v_1 + 2v_2$, the matrix of ϕ in the standard basis is the matrix

$$\phi = \frac{1}{3}\begin{pmatrix} 4\lambda - \mu & 2(\lambda - \mu) \\ 2(\mu - \lambda) & 4\mu - \lambda \end{pmatrix}$$

while the matrix of ψ is the matrix

$$\psi = \frac{1}{3}\begin{pmatrix} A := 4\lambda - \mu & B := 2(\mu - \lambda) \\ C := 2(\lambda - \mu) & D := 4\mu - \lambda \end{pmatrix}$$

The conditions for a free action boil down to:

$$A, B, C, D, A + B, C + D$$

are units in $\mathbb{Z}/n$, and moreover $A \neq B$, $C \neq D, A + B \neq C + D$.

These are in turn equivalent to the condition that

$$\lambda, \mu, \lambda - 4\mu, \lambda - \mu, \mu - 4\lambda, \lambda + 2\mu, 2\lambda + \mu \in (\mathbb{Z}/n)^*.$$

Given $\lambda \in (\mathbb{Z}/n)^*$, consider its direct sum decomposition given by the Chinese remainder theorem and the primary factorization of n. For each prime p dividing n, the residue classes modulo p which are excluded by the above condition are at most five values inside $(\mathbb{Z}/p)^*$, hence we are done if $(\mathbb{Z}/p)^*$ has at least six elements.

Now, since n is relatively prime to 30, each prime number dividing it is greater or equal to $p = 7$.

□

Proposition 11. *Consider the Beauville surface S constructed in* [Bea78], *corresponding to the case $n = 5$.*

Then S is an even surface and $K_S \equiv 2L$, where $H^1(L) = 0$.

Proof. We observe that L is unique, because the torsion group of S is of exponent 5 (see [BC04]).

The existence of L follows exactly as in the proof of the main theorem, where the condition $n \geq 7$ was not used. That $H^1(L) = 0$ follows directly from proposition 3.

□

Acknowledgements. I would like to thank Jonathan Wahl for asking the above question. In the appendix below he describes a construction based on our main result.

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6. Appendix by Jonathan Wahl: A non-$\mathbb{Q}$-Gorenstein Cohen-Macaulay cone X with K_X $\mathbb{Q}$-Cartier

A germ $(X, 0)$ of an isolated normal complex singularity of dimension $n \geq 2$ is called $\mathbb{Q}$-*Gorenstein* if

(1) $(X, 0)$ is Cohen-Macaulay
(2) The dualizing sheaf K_X is $\mathbb{Q}$-Cartier (i.e., the invertible sheaf $\omega_{X-\{0\}}$ has finite order r)
(3) The corresponding cyclic *index one* (or *canonical*) cover $(Y, 0) \to (X, 0)$ is Cohen-Macaulay, hence Gorenstein.

Alternatively, $(X, 0)$ is the quotient of a Gorenstein singularity by a cyclic group acting freely off the singular point. Some early definitions did not require the third condition, which is of course automatic for $n = 2$.

If $(X, 0)$ is $\mathbb{Q}$-Gorenstein, a one-parameter deformation $(\mathcal{X}, 0) \to (\mathbb{C}, 0)$ is called $\mathbb{Q}$-Gorenstein if it is the quotient of a deformation of the index one cover of $(X, 0)$; this is exactly the condition that $(\mathcal{X}, 0)$

is itself $\mathbb{Q}$-Gorenstein. These notions were introduced by Kollár and Shepherd-Barron [2], who made extensive use of the author's explicit smoothings of certain cyclic quotient surface singularities in [3] (5.9); these deformations were patently $\mathbb{Q}$-Gorenstein, and it was important to name this property.

Recently, the author and others considered rational surface singularities admitting a rational homology disk smoothing (i.e., with Milnor number 0). The three-dimensional total space of the smoothing had a rational singularity with K $\mathbb{Q}$-Cartier, but it was not initially clear whether the smoothings were $\mathbb{Q}$-Gorenstein. (This was later established [5] by proving the stronger result that the total spaces were log-terminal.) In fact, one needs to be careful because of the examples of A. Singh:

Example. [4]*: There is a three-dimensional isolated rational (hence Cohen-Macaulay) complex singularity* $(X, 0)$ *with* K_X $\mathbb{Q}$*-Cartier which however is not* $\mathbb{Q}$*-Gorensteiin.*

The purpose of this note is to use F. Catanese's result to provide other examples; they are not rational, but are cones over a smooth projective variety, which could for instance be assumed to be projectively normal with ideal generated by quadrics.

Proposition 12. *Let* S *be a surface as in Theorem 2 of Catanese's paper, with* $h^1(S, \mathcal{O}_S) = 0$*,* L *ample,* $K_S = rL$ *(some* $r > 1$*), and* $h^1(mL) \neq 0$ *for some* $m > 0$*. Let* t *be greater than* r *and relatively prime to it. Then*

(1) *The cone* $R = \mathcal{R}(S, tL) := \oplus_{m\geq 0} H^0(S, \mathcal{O}_S(mtL))$ *is Cohen-Macaulay.*
(2) *The dualizing sheaf of* R *is torsion, of order* t*.*
(3) *The index one cover is* $\mathcal{R}(S, L) := \oplus_{m\geq 0} H^0(S, \mathcal{O}_S(mL))$*, and is not Cohen-Macaulay.*

In particular, R *is not* $\mathbb{Q}$*-Gorenstein.*

Proof. The Cohen-Macaulayness for R follows because $h^1(itL) = 0$, all i, thanks to Kodaira Vanishing. Let $\pi : V \to S$ be the geometric line bundle corresponding to $-tL$; then $H^0(V, \mathcal{O}_V) \equiv R$. Since $K_V \equiv \pi^*(K_S + tL)$, one has that $jK_R \equiv \oplus_{n\in\mathbf{Z}} H^0(S, j(K_S + tL) + nL)$; since $tK_S = r(tL)$ with r and t relatively prime, K_R has order t. Making a cyclic t-fold cover and normalizing gives that $\mathcal{R}(S, L)$ is the index one cover, which as Catanese has noted is not Cohen-Macaulay. □

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Fabrizio Catanese: Lehrstuhl Mathematik VIII, Mathematisches Institut der Universität Bayreuth, NW II, Universitätsstr. 30, 95447 Bayreuth, Germany,
E-mail address: `Fabrizio.Catanese@uni-bayreuth.de`

Jonathan Wahl: Department of Mathematics, University of North Carolina Chapel Hill NC 27599-3250, USA
E-mail address: `jmwahl@email.unc.edu`